Comment on "Eigenfunction expansion of the dyadic Green's function in a gyroelectric chiral medium by cylindrical vector wave functions"

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Several conceptual errors found in the paper by Cheng [Phys. Rev. E **55**, 1950 (1997)] are corrected, and we provide general expansion formulas for the dyadic Green's function using cylindrical vector wave functions, which could be reduced to the exact formulas in special cases (i.e., anisotropic medium, isotropic chiral medium, and isotropic medium). [S1063-651X(99)08403-2]

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Recently, Cheng [1] developed a dyadic Green's function for an unbounded gyroelectric chiral medium by means of an eigenfunction expansion in terms of cylindrical vector wave functions. We report several conceptual mistakes and other errors in Ref. [1] as listed below:

(i) Equation (26) in Ref. [1] is not correct for the case considered in that paper. Mathematically, it is incorrect because the residue value is not correctly estimated; and physically, it is also incorrect because only two poles of the four possible pole points can be taken into the evaluation of the residues [2]. Also, the summation sign corresponding to index j in Cheng's Eqs. (20), (22), (25) and (26) is quite confusing. In fact, k_1, k_3, and k_5 stand for the wave number of three types of left-handed circular polarized waves, while $-k_2, -k_4$, and $-k_6$ denote the wave numbers of the three types of right-handed circularly polarized waves. As obviously seen from the coefficient expressions in the Appendix of Ref. [1], the coefficients k_2, k_4, and k_6 are all negative. Therefore, the summation is never taken.

The solution of an integral similar to Eq. (26) in [1] was evaluated by Engheta and Kowarz [2], Yin and Wang [3], and Li *et al.* [4,5]. Without the summation, a factor 2 from $2\pi i$ should first be given, and the residues of $1/[(\sqrt{k_\rho^2+k_z^2}-k_{1,3,5})(\sqrt{k_\rho^2+k_z^2}-k_{2,4,6})]$ at $k_\rho=\sqrt{k_{1,3,5}^2-k_z^2}$ and $k_z=\sqrt{k_{1,3,5}^2-k_\rho^2}$ should also be correctly estimated. The formulation is detailed below.

Since k_λ (or $k_{\lambda j}$) represents the wave number or propagation constant, k_λ (or $k_{\lambda j}$) satisfying the following equation must be greater than zero:

$$k_\lambda=\sqrt{k_\rho^2+k_z^2},$$

$$k_\lambda \geqslant 0,\ \ 0 \leqslant k_\rho < +\infty \quad \text{and} \quad -\infty < k_z < +\infty.$$

This also ensures the radiation condition in the far zone so that waves propagate away from (not toward) the source (see Ref. [2]). From the denominator of the integral in Eq. (26) of Ref. [1], given as follows, we can determine the pole points:

$$(k_\lambda - k_{1,3,5})(k_\lambda - k_{2,4,6}) = 0,$$

where the subscripts (1,3,5) and (2,4,6) stand for three pairs of equations for point pairs (1,2), (3,4), and (5,6) respectively. Obviously, we need to consider only two pole points (although there exist four possible poles mathematically), i.e.,

$$k_z^u = \sqrt{k_{1,3,5}^2 - k_\rho^2}, \tag{1a}$$

$$k_z^l = -\sqrt{k_{1,3,5}^2 - k_\rho^2}, \tag{1b}$$

where the superscript u denotes that the pole point is located in the upper half sheet of the complex plane, while the superscript l represents that the pole point is located in the lower half sheet. Physically, $k_\lambda - k_{2,4,6}$ does not give any pole point as discussed earlier.

As k_ρ changes from 0 to ∞, we can see physically that k_z^u and k_z^l are almost simultaneously either real numbers or pure imaginary numbers. When we consider the case $z \geqslant z'$ (or $z \leqslant z'$), we need to take the upper point k_z^u (or the lower point k_z^l) into account in the residue evaluation. Therefore, we have

$$\int_{-\infty}^{\infty} dk_z \frac{\boldsymbol{V}_{-n}^{(1)'}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)}{k_\rho(k_\lambda-k_{1,3,5})(k_\lambda-k_{2,4,6})} = \frac{2\pi i k_{1,3,5}}{[k_\rho(k_{1,3,5}-k_{2,4,6})]\sqrt{k_{1,3,5}^2-k_\rho^2}} \times \begin{cases} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_{1,3,5}^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_{1,3,5}^2-k_\rho^2},k_\rho) & \text{for} \quad z \geqslant z' \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_{1,3,5}^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_{1,3,5}^2-k_\rho^2},k_\rho) & \text{for} \quad z \leqslant z'. \end{cases} \tag{2}$$

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In the above integral evaluation, when $z \geqslant z'$, we carried out the counterclockwise integration in the upper sheet of the complex k_z plane so that the value of the integral is the residue itself. However, when $z \leqslant z'$, we conducted the clockwise integration in the lower sheet of the complex k_z plane so that the value of the integral is the negative number of the residue.

(ii) With the mistake in Eq. (26) in Ref. [1], related results have been affected. It is found that the error propagates to Cheng's Eqs. (25a), (25b), and (25d). With this formulation, the correct form of Cheng's Eqs. (25a), (25b), and (25d) are

$$P_{\rho\rho}^{nz} = \frac{\pi i}{k_\rho}\hat{\boldsymbol{\rho}} \cdot \left[\frac{k_1}{(k_1-k_2)\sqrt{k_1^2-k_\rho^2}} \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_1^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_1^2-k_\rho^2},k_\rho) \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_1^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_1^2-k_\rho^2},k_\rho) \end{matrix} \right\} + \frac{k_3}{(k_3-k_4)\sqrt{k_3^2-k_\rho^2}} \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_3^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_3^2-k_\rho^2},k_\rho) \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_3^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_3^2-k_\rho^2},k_\rho) \end{matrix} \right\} \right] \cdot \hat{\boldsymbol{\rho}} \left\{ \begin{matrix} \text{for} \quad z \geqslant z' \\ \text{for} \quad z \leqslant z'. \end{matrix} \right. \tag{3a}$$

$$P_{\phi\rho}^{nz} = -\frac{\pi}{k_\rho}\hat{\boldsymbol{\phi}} \cdot \left[\frac{k_1}{(k_1-k_2)\sqrt{k_1^2-k_\rho^2}} \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_1^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_1^2-k_\rho^2},k_\rho) \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_1^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_1^2-k_\rho^2},k_\rho) \end{matrix} \right\} - \frac{k_3}{(k_3-k_4)\sqrt{k_3^2-k_\rho^2}} \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_3^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_3^2-k_\rho^2},k_\rho) \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_3^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_3^2-k_\rho^2},k_\rho) \end{matrix} \right\} \right] \cdot \hat{\boldsymbol{\rho}} \left\{ \begin{matrix} \text{for} \quad z \geqslant z' \\ \text{for} \quad z \leqslant z'. \end{matrix} \right. \tag{3b}$$

$$P_{zz}^{nz} = \frac{2\pi i k_5}{k_\rho(k_5-k_6)\sqrt{k_5^2-k_\rho^2}} \hat{\boldsymbol{z}} \cdot \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-\sqrt{k_5^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(\sqrt{k_5^2-k_\rho^2},k_\rho) \\ \boldsymbol{V}_{-n}^{(1)'}(\sqrt{k_5^2-k_\rho^2},k_\rho)\boldsymbol{V}_n^{(1)}(-\sqrt{k_5^2-k_\rho^2},k_\rho) \end{matrix} \right\} \cdot \hat{\boldsymbol{z}} \left\{ \begin{matrix} \text{for} \quad z \geqslant z' \\ \text{for} \quad z \leqslant z'. \end{matrix} \right. \tag{3c}$$

(iii) In a similar fashion, the integration carried out in Eq. (20) in Ref. [1] is also found incorrect. The correct form of Eq. (20) in Ref. [1] should read

$$\int_0^\infty dk_\rho \frac{\boldsymbol{V}_{-n}^{(1)'}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)}{k_\rho(k_\lambda-k_{1,3,5})(k_\lambda-k_{2,4,6})} = -\frac{1}{|n|(k_z-k_{1,3,5})(k_z-k_{2,4,6})} \boldsymbol{T}_v \boldsymbol{T}_{v'} \left[\left(\frac{\rho_<}{\rho_>} \right)^{|n|} \right] e^{i[n(\phi^> - \phi^<) + k_z(z^> - z^<)]} + \frac{2\pi i k_{1,3,5}}{(k_{1,3,5}-k_{2,4,6})(k_{1,3,5}^2-k_z^2)} \left\{ \begin{matrix} \boldsymbol{V}_{-n}^{(1)'}(-k_z,\sqrt{k_{1,3,5}^2-k_z^2})\boldsymbol{V}_n^{(3)}(k_z,\sqrt{k_{1,3,5}^2-k_z^2}) & \text{for} \quad \rho \geqslant \rho' \\ \boldsymbol{V}_{-n}^{(3)'}(-k_z,\sqrt{k_{1,3,5}^2-k_z^2})\boldsymbol{V}_n^{(1)}(k_z,\sqrt{k_{1,3,5}^2-k_z^2}) & \text{for} \quad \rho \leqslant \rho', \end{matrix} \right. \tag{4}$$

where the following relation has been utilized:

$$\lim_{k_\rho \to 0} [J_n(k_\rho \rho^<) Y_n(k_\rho \rho^>)] = -\frac{1}{|n|\pi} \left(\frac{\rho_<}{\rho_>} \right)^{|n|}.$$

Therefore, the vector operators $\boldsymbol{T}_{\substack{v\\w}}$ and $\boldsymbol{T}_{\substack{v'\\w'}}$ that are not defined in Ref. [1] should be specified as

$$\boldsymbol{T}_{\substack{v\\w}}(\bullet) = \frac{1}{\sqrt{2}} \left[\boldsymbol{\nabla} \times (\bullet \hat{\boldsymbol{z}}) \pm \frac{1}{k_z} \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\bullet \hat{\boldsymbol{z}}) \right], \tag{5a}$$

$$\boldsymbol{T}_{\substack{v'\\w'}}(\bullet) = \frac{1}{\sqrt{2}} \left[\boldsymbol{\nabla}' \times (\bullet \hat{\boldsymbol{z}}) \pm \frac{1}{k_z} \boldsymbol{\nabla}' \times \boldsymbol{\nabla}' \times (\bullet \hat{\boldsymbol{z}}) \right], \tag{5b}$$

where the prime denotes the $\boldsymbol{\nabla}'$ operation with respect to the coordinates (ρ',ϕ',z'). The operators in the final expressions are necessarily provided here so that readers can make use of them straightforwardly. Without explicit expressions for the operators, readers have to reconsider the problem and derive these themselves.

(iv) Due to the change of Eq. (20), Eqs. (22a), (22b), and (22d) in Ref. [1] should read as follows:

$$\begin{bmatrix} P_{\rho\rho}^{n\rho} \\ P_{\phi\phi}^{n\rho} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \end{bmatrix} \cdot \left(-\left[\frac{1}{(k_z-k_1)(k_z-k_2)} + \frac{1}{(k_z-k_3)(k_z-k_4)} \right] \frac{\boldsymbol{T}_v \boldsymbol{T}_{v'}}{2|n|} \left[\left(\frac{\rho_<}{\rho_>} \right)^{|n|} e^{i[n(\phi^> - \phi^<) + k_z(z^> - z^<)]} \right] + \pi i \right.$$
$$\left. \times \begin{Bmatrix} \left[\frac{\boldsymbol{V}_{-n}^{(1)\prime}(-k_z, \sqrt{k_1^2-k_z^2}) \boldsymbol{V}_n^{(3)}(k_z, \sqrt{k_1^2-k_z^2})}{(1-k_2/k_1)(k_1^2-k_z^2)} + \frac{\boldsymbol{V}_{-n}^{(1)\prime}(-k_z, \sqrt{k_3^2-k_z^2}) \boldsymbol{V}_n^{(3)}(k_z, \sqrt{k_3^2-k_z^2})}{(1-k_4/k_3)(k_3^2-k_z^2)} \right] \\ \left[\frac{\boldsymbol{V}_{-n}^{(3)\prime}(-k_z, \sqrt{k_1^2-k_z^2}) \boldsymbol{V}_n^{(1)}(k_z, \sqrt{k_1^2-k_z^2})}{(1-k_2/k_1)(k_1^2-k_z^2)} + \frac{\boldsymbol{V}_{-n}^{(3)\prime}(-k_z, \sqrt{k_3^2-k_z^2}) \boldsymbol{V}_n^{(1)}(k_z, \sqrt{k_3^2-k_z^2})}{(1-k_4/k_3)(k_3^2-k_z^2)} \right] \end{Bmatrix} \right) \cdot \begin{bmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \end{bmatrix} \begin{cases} \text{for} & \rho \geqslant \rho' \\ \text{for} & \rho \leqslant \rho', \end{cases} \tag{6a}$$

$$\begin{bmatrix} P_{\phi\rho}^{n\rho} \\ -P_{\rho\phi}^{n\rho} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{\rho}} \end{bmatrix} \cdot \left(-\left[\frac{1}{(k_z-k_1)(k_z-k_2)} - \frac{1}{(k_z-k_3)(k_z-k_4)} \right] \frac{i\boldsymbol{T}_v \boldsymbol{T}_{v'}}{2|n|} \left[\left(\frac{\rho_<}{\rho_>} \right)^{|n|} e^{i[n(\phi^> - \phi^<) + k_z(z^> - z^<)]} \right] - \pi \right.$$
$$\left. \times \begin{Bmatrix} \left[\frac{\boldsymbol{V}_{-n}^{(1)\prime}(-k_z, \sqrt{k_1^2-k_z^2}) \boldsymbol{V}_n^{(3)}(k_z, \sqrt{k_1^2-k_z^2})}{(1-k_2/k_1)(k_1^2-k_z^2)} - \frac{\boldsymbol{V}_{-n}^{(1)\prime}(-k_z, \sqrt{k_3^2-k_z^2}) \boldsymbol{V}_n^{(3)}(k_z, \sqrt{k_3^2-k_z^2})}{(1-k_4/k_3)(k_3^2-k_z^2)} \right] \\ \left[\frac{\boldsymbol{V}_{-n}^{(3)\prime}(-k_z, \sqrt{k_1^2-k_z^2}) \boldsymbol{V}_n^{(1)}(k_z, \sqrt{k_1^2-k_z^2})}{(1-k_2/k_1)(k_1^2-k_z^2)} - \frac{\boldsymbol{V}_{-n}^{(3)\prime}(-k_z, \sqrt{k_3^2-k_z^2}) \boldsymbol{V}_n^{(1)}(k_z, \sqrt{k_3^2-k_z^2})}{(1-k_4/k_3)(k_3^2-k_z^2)} \right] \end{Bmatrix} \right)$$
$$\cdot \begin{bmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \end{bmatrix} \begin{cases} \text{for} & \rho \geqslant \rho' \\ \text{for} & \rho \leqslant \rho', \end{cases} \tag{6b}$$

$$P_{zz}^{n\rho} = \hat{\boldsymbol{z}} \cdot \left(-\frac{\boldsymbol{T}_v \boldsymbol{T}_{v'}}{|n|(k_z-k_5)(k_z-k_6)} \left[\left(\frac{\rho_<}{\rho_>} \right)^{|n|} e^{i[n(\phi^> - \phi^<) + k_z(z^> - z^<)]} \right] + \frac{2\pi i k_5}{(k_5-k_6)(k_5^2-k_z^2)} \right.$$
$$\left. \times \begin{Bmatrix} [\boldsymbol{V}_{-n}^{(1)\prime}(-k_z, \sqrt{k_5^2-k_z^2}) \boldsymbol{V}_n^{(3)}(k_z, \sqrt{k_5^2-k_z^2})] \\ [\boldsymbol{V}_{-n}^{(3)\prime}(-k_z, \sqrt{k_5^2-k_z^2}) \boldsymbol{V}_n^{(1)}(k_z, \sqrt{k_5^2-k_z^2})] \end{Bmatrix} \right) \cdot \hat{\boldsymbol{z}} \begin{cases} \text{for} & \rho \geqslant \rho' \\ \text{for} & \rho \leqslant \rho'. \end{cases} \tag{6c}$$

Previously, it was claimed below Eq. (26) in Ref. [1] that ''the components arising in Eqs. (24a) and (24b) can be reduced to the counterparts of chiral media.'' In fact, this is not true simply because the correct form of the dyadic Green's functions in Secs. IV A and IV B of Ref. [1] consists of additional factors $k_j/\sqrt{k_j^2-k_\rho^2}$ and $k_j/\sqrt{k_j^2-k_z^2}$ which the forms given by Cheng [1] do not contain.

Now the above equations for the gyroelectric chiral medium can be systematically reduced to those for the anisotropic medium ($\xi_c=0$), the isotropic chiral medium ($g=0$), or the isotropic medium (both $\xi_c=0$ and $g=0$). It has been confirmed that exactly the same results for those special cases can be obtained from the reduction.

(v) In the previous considerations, it has been assumed that Eq. (21), together with Eqs. (22a)–(22d) and Eq. (24a), together with Eqs. (25a)–(25d) in Ref. [1] are correct. However, they are incorrect. According to Cheng's Eq. (19), we have

$$\overline{\mathbf{P}}^{n\rho} = \int_0^\infty \frac{dk_\rho}{k_\rho} \overline{\mathbf{A}}^{-1} \cdot \boldsymbol{V}_{-n}^{(1)\prime}(-k_z, k_\rho) \boldsymbol{V}_n^{(1)}(k_z, k_\rho)$$
$$= \begin{bmatrix} \mathcal{P}_{\rho\rho}^{n\rho} & \mathcal{P}_{\rho\phi}^{n\rho} & \mathcal{P}_{\rho z}^{n\rho} \\ \mathcal{P}_{\phi\rho}^{n\rho} & \mathcal{P}_{\phi\phi}^{n\rho} & \mathcal{P}_{\phi z}^{n\rho} \\ \mathcal{P}_{z\rho}^{n\rho} & \mathcal{P}_{z\phi}^{n\rho} & \mathcal{P}_{zz}^{n\rho} \end{bmatrix}. \tag{7}$$

The inverse matrix $\overline{\mathbf{A}}^{-1}$ is given by

$$\overline{\mathbf{A}}^{-1}=\begin{bmatrix} A_{\rho\rho} & -A_{\rho\phi} & 0 \\ A_{\rho\phi} & A_{\phi\phi} & 0 \\ 0 & 0 & A_{zz} \end{bmatrix} \quad (8)$$

where the nonzero elements $A_{\rho\rho}, A_{\phi\rho}, A_{\phi\phi}$, and A_{zz} are given in the Appendix of Ref. [1]. At the same time, the dyadic $\boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)$ is given by

$$\boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)=\begin{bmatrix} V'_\rho V_\rho & V'_\rho V_\phi & V'_\rho V_z \\ V'_\phi V_\rho & V'_\phi V_\phi & V'_\phi V_z \\ V'_z V_\rho & V'_z V_\phi & V'_z V_z \end{bmatrix}. \quad (9)$$

Therefore, we have the $\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}}$ component of $\overline{\mathbf{P}}$ as follows:

$$\begin{aligned}\overline{\mathbf{P}}\big|_{\rho\rho}&=\int_0^\infty \frac{dk_\rho}{k_\rho}(A_{\rho\rho}V'_\rho V_\rho - A_{\rho\phi}V'_\phi V_\rho)\\ &=\int_0^\infty \frac{dk_\rho}{k_\rho}[A_{\rho\rho}\hat{\boldsymbol{\rho}}\cdot \boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)\cdot\hat{\boldsymbol{\rho}}-A_{\rho\phi}\hat{\boldsymbol{\rho}}\cdot \boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)\cdot\hat{\boldsymbol{\phi}}]. \quad (10)\end{aligned}$$

It is clearly seen from the above form that the correct $\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}}$ component $\mathcal{P}_{\rho\rho}$ has one more term as compared with the $\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}}$ component $P_{\rho\rho}$ given in Ref. [1]. The similar conclusion applies to other elements of the dyadic $\overline{\mathbf{P}}$, e.g., instead of the zero component given in Eq. (22c) of Ref. [1], the correct $\hat{\boldsymbol{\phi}}\hat{\boldsymbol{z}}$ component $\mathcal{P}_{\phi z}$ should be

$$\begin{aligned}\overline{\mathbf{P}}\big|_{\phi z}&=\int_0^\infty \frac{dk_\rho}{k_\rho}(A_{\rho\phi}V'_\rho V_z + A_{\phi\phi}V'_\phi V_z)\\ &=\int_0^\infty \frac{dk_\rho}{k_\rho}[A_{\rho\phi}\hat{\boldsymbol{\rho}}\cdot \boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)\cdot\hat{\boldsymbol{z}}+A_{\phi\phi}\hat{\boldsymbol{\phi}}\cdot \boldsymbol{V}_{-n}^{(1)\prime}(-k_z,k_\rho)\boldsymbol{V}_n^{(1)}(k_z,k_\rho)\cdot\hat{\boldsymbol{z}}]. \quad (11)\end{aligned}$$

The final and correct form of the dyadics $\overline{\boldsymbol{P}}$ in Eq. (21) and $\overline{\boldsymbol{Q}}$ in Eq. (23) of Ref. [1] are obtained and given as follows:

$$\overline{\boldsymbol{P}}=\overline{\boldsymbol{P}}_{1+}+\overline{\boldsymbol{P}}_{3+}+\overline{\boldsymbol{P}}_{5+}, \quad (12a)$$

$$\overline{\boldsymbol{Q}}=\overline{\boldsymbol{Q}}_{1-}+\overline{\boldsymbol{Q}}_{3-}+\overline{\boldsymbol{Q}}_{5-}, \quad (12b)$$

where $\overline{\boldsymbol{P}}_{j+}$ for a given $k_{\lambda j+}$ is given by

$$\begin{aligned}\overline{\boldsymbol{P}}_{j+}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dh\sum_{n=-\infty}^{\infty}\Bigg[&\overline{\boldsymbol{P}}_{a,j+}^{n\rho}\cdot\frac{1}{|n|}\boldsymbol{T}_v\boldsymbol{T}_{v'}\left(\frac{\rho_<}{\rho_>}\right)^{|n|}e^{i[n(\phi^>-\phi^<)+h(z^>-z^<)]}\\ &+\overline{\boldsymbol{P}}_{b,j+}^{n\rho}\cdot\begin{cases}\boldsymbol{V}_n^{(1)}(k_z,k_{\lambda j+})\boldsymbol{V}'_{-n}(-k_z,k_{\lambda j+}), & \rho>\rho'\\ \boldsymbol{V}_{-n}(-k_z,k_{\lambda j+})\boldsymbol{V}'^{(1)}_n(k_z,k_{\lambda j+}), & \rho<\rho'\end{cases}\Bigg], \quad (13a)\end{aligned}$$

while $\overline{\boldsymbol{Q}}_{j-}$ for a given $k_{\lambda j-}$ is represented as

$$\begin{aligned}\overline{\boldsymbol{Q}}_{j-}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}\Bigg[&\overline{\boldsymbol{Q}}_{a,j-}^{n\rho}\cdot\frac{1}{|n|}\boldsymbol{T}_w\boldsymbol{T}_{w'}\left(\frac{\rho_<}{\rho_>}\right)^{|n|}e^{i[n(\phi^>-\phi^<)+k_z(z^>-z^<)]}\\ &+\overline{\boldsymbol{Q}}_{b,j-}^{n\rho}\cdot\begin{cases}\boldsymbol{W}_n^{(1)}(k_z,k_{\lambda j-})\boldsymbol{W}'_{-n}(-k_z,k_{\lambda j-}), & \rho>\rho'\\ \boldsymbol{W}_{-n}(-k_z,k_{\lambda j-})\boldsymbol{W}'^{(1)}_n(k_z,k_{\lambda j-}), & \rho<\rho'\end{cases}\Bigg]. \quad (13b)\end{aligned}$$

In Eqs. (13a) and (13b), the intermediate dyads are defined as

$$\left.\begin{array}{l}\overline{\boldsymbol{P}}_{a,1+}^{n\rho}\\ \overline{\boldsymbol{Q}}_{a,1-}^{n\rho}\end{array}\right\}=-\frac{\pi i}{2(h-k_{1,\pm})(h-k_{2,\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}}+\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}})+\frac{\pi}{2(h-k_{1,\pm})(h-k_{2,\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\rho}}-\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\phi}}), \quad (14a)$$

$$\left.\begin{array}{l}\overline{\boldsymbol{P}}_{a,3+}^{n\rho}\\ \overline{\boldsymbol{Q}}_{a,3-}^{n\rho}\end{array}\right\}=-\frac{\pi i}{2(h-k_{3,\pm})(h-k_{4,\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}}+\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}})-\frac{\pi}{2(h-k_{3,\pm})(h-k_{4,\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\rho}}-\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\phi}}), \quad (14b)$$

$$\left.\begin{array}{l}\bar{\boldsymbol{P}}^{n\rho}_{a,5+}\\ \bar{\boldsymbol{Q}}^{n\rho}_{a,5-}\end{array}\right\} = -\pi i \frac{1}{(h-k_{5,\pm})(h-k_{6,\pm})}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}, \tag{14c}$$

$$\left.\begin{array}{l}\bar{\boldsymbol{P}}^{nz}_{b,1+}\\ \bar{\boldsymbol{Q}}^{nz}_{b,1-}\end{array}\right\} = \frac{\pi i}{2}\frac{k_{1\pm}}{\lambda^2_{1\pm}(k_{1\pm}-k_{2\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}}+\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}}) - \frac{\pi}{2}\frac{k_{1\pm}}{\lambda^2_{1\pm}(k_{1\pm}-k_{2\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\rho}}-\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\phi}}), \tag{14d}$$

$$\left.\begin{array}{l}\bar{\boldsymbol{P}}^{nz}_{b,3+}\\ \bar{\boldsymbol{Q}}^{nz}_{b,3-}\end{array}\right\} = \frac{\pi i}{2}\frac{k_{3\pm}}{\lambda^2_{3\pm}(k_{3\pm}-k_{4\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\phi}}+\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\rho}}) + \frac{\pi}{2}\frac{k_{3\pm}}{\lambda^2_{3\pm}(k_{3\pm}-k_{4\pm})}(\hat{\boldsymbol{\phi}}\hat{\boldsymbol{\rho}}-\hat{\boldsymbol{\rho}}\hat{\boldsymbol{\phi}}), \tag{14e}$$

$$\left.\begin{array}{l}\bar{\boldsymbol{P}}^{nz}_{b,5+}\\ \bar{\boldsymbol{Q}}^{nz}_{b,5-}\end{array}\right\} = \pi i \frac{k_{5\pm}}{\lambda^2_{5\pm}(k_{5\pm}-k_{6\pm})}(\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}). \tag{14f}$$

In the above-given and subsequent expressions, it is assumed that

$$\begin{bmatrix} k_{\lambda j+} \\ k_{j+} \end{bmatrix} = \begin{bmatrix} k_{\lambda j} \\ k_j \end{bmatrix}, \tag{15a}$$

$$\begin{bmatrix} k_{\lambda j-} \\ k_{j-} \end{bmatrix} = \begin{bmatrix} k_{\lambda j} \\ k_j \end{bmatrix}_{\xi_c \to -\xi}, \tag{15b}$$

where the wave numbers $k_j (j=1-6)$ are given by Eqs. (A6)–(A11) in Ref. [1].

It is now seen that Eqs. (22a)–(22c) of Ref. [1] are only part of the complete expression given as above. The formulations of Eqs. (24a) and (24b), together with Eqs. (25a)–(25c), in Ref. [1], can be made in a similar fashion. However, they are not provided here because Eqs. (19) and (23) of Ref. [1] are actually incorrect. The mistakes in Eqs. (19) and (23) of Ref. [1] are due to the use of the following incorrect formula:

$$\nabla \times [\boldsymbol{a}\boldsymbol{M}] = \boldsymbol{a}\nabla \times \boldsymbol{M},$$

since

$$\sum_i \sum_j a_i [\nabla M_j \times \hat{\boldsymbol{x}}_i]\hat{\boldsymbol{x}}_j \neq \sum_i \sum_j a_i \hat{\boldsymbol{x}}_i [\nabla M_j \times \hat{\boldsymbol{x}}_j],$$

where $\boldsymbol{a}$ stands for a constant vector and $\boldsymbol{M}$ denotes the vector wave function.

(iv) Formula (4) in Ref. [1] is incorrect, since the last term in that form of the left-handed side is not obtainable after Eq. (3) is substituted into Eq. (2). Obviously, this can be seen from the relation given by

$$\bar{\boldsymbol{\epsilon}} \cdot [\boldsymbol{J}(\boldsymbol{r}') \cdot \bar{\boldsymbol{\Gamma}}(\boldsymbol{r},\boldsymbol{r}')] \neq \boldsymbol{J}(\boldsymbol{r}') \cdot [\bar{\boldsymbol{\epsilon}} \cdot \bar{\boldsymbol{\Gamma}}(\boldsymbol{r},\boldsymbol{r}')],$$

where $\bar{\boldsymbol{\epsilon}}$, $\boldsymbol{J}(\boldsymbol{r}')$, and $\bar{\boldsymbol{\Gamma}}(\boldsymbol{r},\boldsymbol{r}')$ identify the permittivity tensor, the vector electric current distribution, and electric dyadic Green's function, respectively. Only when the left- and right-handed sides of this form are the same, will the results obtained subsequently in Ref. [1] be valid. However, the author of Ref. [1] ignored the condition, so that part of his results, even though correctly formed somewhere, are very limited in their usefulness since the aforementioned condition is rarely satisfied in practical problems [6–9].

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